

Introduction to Statistics

Math 1040

Sample Exam II – Chapters 5-7

4 Problem Pages – 4 Formula/Table Pages

Time Limit: 90 Minutes¹ No Scratch Paper Calculator Allowed: Scientific

Name: _____

The point value of each problem is in the left-hand margin. You must show your work to receive any credit, except on problems 1 & 2. Work neatly.

(6) 1. Fill in the blanks.

- (a) The probability of a ball chosen in random, from a collection of 7 balls of which exactly 4 are red, being red is _____.
- (b) The value of ${}_8P_5$ is _____.
- (c) If Z is a standard normal random variable, then $P(Z > -1.52) =$ _____.

(6) 2. True or False.

- () (a) If Z is a standard normal variable and $P(Z < z_0) = 0.747$, then $z_0 = 0.665$.
- () (b) Suppose a fair coin is flipped 101 times. Let A be the event that each of the first 100 tosses results in a head. Let B be the event that the 101st toss results in a tail. Events A and B are independent.
- () (c) Suppose a fair coin is flipped 3 times. Let E be the event that two heads are observed. Let F be the event that at least one tail is observed. Events E and F are mutually exclusive.

(8) 3. Consider a group of 6 people.

- (a) In how many ways can they be arranged in a row?

- (b) In how many ways can we pick a committee of size 3 from this group?

- (c) In how many ways can a president and a vice-president be chosen from this group?

¹If you exceed the time limit, you will receive a score of zero.

(5) 4. How many different eight-letter passwords can be formed from the letters in the word ENGINEER?

(6) 5. What is the probability of getting a spade flush from a standard 52-card deck of cards? (A standard deck consists of 13 hearts, diamonds, clubs and spades. A spade flush is achieved by being dealt, without replacement, 5 consecutive spade cards.) Note: You must explain your work through words and/or formula(s).

(6) 6. Let X be a discrete random variable with the probability $P(X = x) = P(x) = \begin{cases} \frac{1}{8} & , \text{if } x = 1, 2 \\ \frac{1}{4} & , \text{if } x = 3 \\ \frac{1}{2} & , \text{if } x = 4 \end{cases}$.

Find the expected value or mean of X , μ_x , and the variance of X , σ_x .

- (6) 7. A bag contains 5 red balls numbered one through five and 6 blue balls numbered one through six. Suppose three balls are chosen in random with each ball being replaced before the next selection is made. What is the probability that the first ball is a one, the second ball is a red and the third ball is a six? Note: You must explain your work through words and/or formula(s).
- (6) 8. A bag contains 5 red balls numbered one through five and 6 blue balls numbered one through six. What is the probability that a ball chosen in random is a one or blue? Note: You must explain your work through words and/or formula(s).
- (8) 9. Suppose in the united States 60% of murders are committed with a firearm. Suppose 15 murders are randomly selected. Find the probability that between 10 and 12 murders, inclusive, are committed with a firearm. Show your work.

- (6) 10. Suppose 5% of college students are not punctual. For a Math 1040 class with 35 students, let the random variable X represent the number of students in this class which are late to class. Find the mean of X , the average number students late to class, and the standard deviation of X .
- (6) 11. The length of human pregnancies is normally distributed with mean $\mu = 266$ days and standard deviation $\sigma = 16$ days. What percent of pregnancies last between 242 and 262 days. Suppose an unusually long pregnancy is the one that is in the top 2%. Determine the length of pregnancy that separates an unusually long pregnancy from one that is not unusually long (98th percentile).
- (6) 12. Studies show that gasoline use for compact cars sold in the United States is normally distributed, with mean of 25.5 miles per gallon (mpg) and a standard deviation of 4.5 mpg. If a manufacturer wishes to develop a compact car that outperforms 95% of the current compacts in fuel economy, what must the gasoline use rate for the new car be?

For independent events E and F , $P(E \text{ and } F) = P(E)P(F)$

$$P(E \text{ or } F) = P(E) + P(F) - P(E \text{ and } F)$$

For mutually exclusive (disjoint) events E and F , $P(E \text{ or } F) = P(E) + P(F)$

$${}_n P_r = \frac{n!}{(n-r)!}, \quad {}_n C_r = \frac{n!}{r!(n-r)!}$$

$$\mu_x = \sum x P(x), \quad \sigma_x^2 = \sum (x - \mu_x)^2 P(x) = \sum x^2 P(x) - \mu_x^2$$

$$P(X = x) = P(x) = {}_n C_x p^x (1-p)^{n-x}, \quad \mu_x = np, \quad \sigma_x^2 = np(1-p)$$

$$Z = \frac{X - \mu}{\sigma}$$

Cumulative Binomial Probability Distribution

This table computes the cumulative probability of obtaining x successes in n trials of a binomial experiment with probability of success p .

Table IV (continued)

n	x	0.01	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	
15	0	0.8601	0.4633	0.2059	0.0874	0.0352	0.0134	0.0047	0.0016	0.0005	0.0001	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+
	1	0.9904	0.8290	0.5490	0.3186	0.1671	0.0802	0.0353	0.0142	0.0052	0.0017	0.0005	0.0001	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+
	2	0.9996	0.9638	0.8159	0.6042	0.3980	0.2361	0.1268	0.0617	0.0271	0.0107	0.0037	0.0011	0.0003	0.0001	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+
	3	1.0000-	0.9945	0.9444	0.8227	0.6482	0.4613	0.2969	0.1727	0.0905	0.0424	0.0176	0.0063	0.0019	0.0005	0.0001	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+
	4	1.0000-	0.9994	0.9873	0.9383	0.8358	0.6865	0.5155	0.3519	0.2173	0.1204	0.0592	0.0255	0.0093	0.0028	0.0007	0.0001	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+
	5	1.0000-	0.9999	0.9978	0.9832	0.9389	0.8518	0.7216	0.5643	0.4032	0.2608	0.1509	0.0769	0.0338	0.0124	0.0037	0.0008	0.0001	0.0000+	0.0000+	0.0000+	0.0000+
	6	1.0000-	1.0000-	0.9997	0.9964	0.9819	0.9434	0.8689	0.7548	0.6098	0.4522	0.3036	0.1818	0.0950	0.0422	0.0152	0.0042	0.0008	0.0001	0.0000+	0.0000+	0.0000+
	7	1.0000-	1.0000-	1.0000-	0.9994	0.9958	0.9827	0.9500	0.8868	0.7869	0.6535	0.5000	0.3465	0.2131	0.1132	0.0500	0.0173	0.0042	0.0006	0.0000+	0.0000+	0.0000+
	8	1.0000-	1.0000-	1.0000-	0.9999	0.9992	0.9958	0.9848	0.9578	0.9050	0.8182	0.6964	0.5478	0.3902	0.2452	0.1311	0.0566	0.0181	0.0036	0.0003	0.0000+	0.0000+
	9	1.0000-	1.0000-	1.0000-	1.0000-	0.9999	0.9992	0.9963	0.9876	0.9662	0.9231	0.8491	0.7392	0.5968	0.4357	0.2784	0.1484	0.0611	0.0168	0.0022	0.0001	0.0001
	10	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	0.9999	0.9993	0.9972	0.9907	0.9745	0.9408	0.8796	0.7827	0.6481	0.4845	0.3135	0.1642	0.0617	0.0127	0.0006	0.0006
	11	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	0.9999	0.9995	0.9981	0.9937	0.9824	0.9576	0.9095	0.8273	0.7031	0.5387	0.3518	0.1773	0.0556	0.0055	0.0055
	12	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	0.9999	0.9997	0.9989	0.9963	0.9893	0.9729	0.9383	0.8732	0.7639	0.6020	0.3958	0.1841	0.0362	0.0362
	13	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	0.9999	0.9995	0.9983	0.9948	0.9858	0.9647	0.9198	0.8529	0.6814	0.4510	0.1710	0.1710
	14	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	0.9999	0.9995	0.9984	0.9953	0.9866	0.9648	0.9126	0.7941	0.5367	0.5367
	15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
20	0	0.8179	0.3585	0.1216	0.0388	0.0115	0.0032	0.0008	0.0002	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+
	1	0.9831	0.7358	0.3917	0.1756	0.0692	0.0243	0.0076	0.0021	0.0005	0.0001	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+
	2	0.9990	0.9245	0.6769	0.4049	0.2061	0.0913	0.0355	0.0121	0.0036	0.0009	0.0002	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+
	3	1.0000-	0.9841	0.8670	0.6477	0.4114	0.2252	0.1071	0.0444	0.0160	0.0049	0.0013	0.0003	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+
	4	1.0000-	0.9974	0.9568	0.8298	0.6296	0.4148	0.2375	0.1182	0.0510	0.0189	0.0059	0.0015	0.0003	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+
	5	1.0000-	0.9997	0.9887	0.9327	0.8042	0.6172	0.4164	0.2454	0.1256	0.0553	0.0207	0.0064	0.0016	0.0003	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+
	6	1.0000-	1.0000-	0.9976	0.9781	0.9133	0.7858	0.6080	0.4166	0.2500	0.1299	0.0577	0.0214	0.0065	0.0015	0.0003	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+
	7	1.0000-	1.0000-	0.9996	0.9941	0.9679	0.8982	0.7723	0.6010	0.4159	0.2520	0.1316	0.0580	0.0210	0.0060	0.0013	0.0002	0.0000+	0.0000+	0.0000+	0.0000+	0.0000+
	8	1.0000-	1.0000-	0.9999	0.9987	0.9900	0.9591	0.8867	0.7624	0.5956	0.4143	0.2517	0.1308	0.0565	0.0196	0.0051	0.0009	0.0001	0.0000+	0.0000+	0.0000+	0.0000+
	9	1.0000-	1.0000-	1.0000-	0.9998	0.9974	0.9861	0.9520	0.8782	0.7553	0.5914	0.4119	0.2493	0.1275	0.0532	0.0171	0.0039	0.0006	0.0000+	0.0000+	0.0000+	0.0000+
	10	1.0000-	1.0000-	1.0000-	1.0000-	0.9994	0.9961	0.9829	0.9468	0.8725	0.7507	0.5881	0.4086	0.2447	0.1218	0.0480	0.0139	0.0026	0.0002	0.0000+	0.0000+	0.0000+
	11	1.0000-	1.0000-	1.0000-	1.0000-	0.9999	0.9991	0.9949	0.9804	0.9435	0.8692	0.7483	0.5857	0.4044	0.2376	0.1133	0.0409	0.0100	0.0013	0.0001	0.0000+	0.0000+
	12	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	0.9998	0.9987	0.9940	0.9790	0.9420	0.8684	0.7480	0.5841	0.3990	0.2277	0.1018	0.0321	0.0059	0.0004	0.0000+	0.0000+
	13	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	0.9997	0.9985	0.9935	0.9786	0.9423	0.8701	0.7500	0.5834	0.3920	0.2142	0.0867	0.0219	0.0024	0.0000+	0.0000+
	14	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	0.9997	0.9984	0.9936	0.9793	0.9447	0.8744	0.7546	0.5836	0.3828	0.1958	0.0673	0.0113	0.0003	0.0003
	15	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	0.9997	0.9985	0.9941	0.9811	0.9490	0.8818	0.7625	0.5852	0.3704	0.1702	0.0432	0.0026	0.0026
	16	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	0.9997	0.9987	0.9951	0.9840	0.9556	0.8929	0.7748	0.5886	0.3523	0.1330	0.0159	0.0159
	17	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	0.9998	0.9998	0.9991	0.9964	0.9879	0.9645	0.9087	0.7939	0.5951	0.3231	0.0755	0.0755
	18	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	0.9999	0.9995	0.9979	0.9924	0.9757	0.9308	0.8244	0.6083	0.2642	0.2642
	19	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	1.0000-	0.9999	0.9992	0.9968	0.9885	0.9612	0.8784	0.6415	0.6415
	20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Note: 0.0000+ means the probability is 0.0000 rounded to four decimal places. However, the probability is *not* zero. 1.0000- means the probability is 1.0000 rounded to four decimal places. However, the probability is *not* one.

Introduction to Statistics

Math 1040

Additional (not on the Sample Exam) Review Problems for Chapters 5-7

This is not an exhaustive list of all possible types of problems.

1. Fill in the blanks.

- (a) An _____ is any collection of outcomes from a probability experiment.
- (b) If E and F are disjoint events, then $P(E \text{ or } F) =$ _____.
- (c) There are _____ ways to pick a committee of size 3 from a group of size 8.
- (d) The probability of obtaining three heads in a row when flipping a fair coin is _____.
- (e) If E and F are independent events, then $P(E \text{ and } F) =$ _____.
- (f) If an experiment has three disjoint outcomes A , B and C with $P(A) = 0.2$ and $P(B) = 0.5$, then $P(C) =$ _____.
- (g) If Z is a standard normal random variable, then $P(Z > 0.25) =$ _____.

2. True or False.

- () (a) If, based on empirical results, the probability of an event is approximated to be zero, then that event will never occur.
- () (b) ${}_8C_3 = {}_8C_5$.
- () (c) $P(-0.25 < Z < 1.03) = P(-0.25 \leq Z \leq 1.03)$.
- () (d) In a uniform distribution, the value of the probability density function is constant.
- () (e) $P(E) + P(E^c) = 1$.

3. Consider a group of 7 people.

- (a) In how many ways can they be arranged in a row?
- (b) In how many ways can we pick a committee of size 5 from this group?
- (c) In how many ways can a president, a vice-president, and a treasurer be chosen from this group?

4. An unbalanced six-sided die is rolled 125 times. The number of times, frequency, it lands on numbers one through five is listed below.

Die landing on	1	2	3	4	5	6
Frequency	?	24	20	16	27	23

- (a) How many times did the die land on number one?
- (b) Based on this data, estimate the probability that on the next roll the die lands on one.
- (c) If in 1000 rolls, the die lands on number one 150 times, give a better estimate of the probability that on the next roll the die lands on one.

5. According to the U.S. Census Bureau, the probability that a randomly selected household speaks only English at home is 0.81. The probability that a randomly selected household speaks only Spanish at home is 0.12.
- (a) What is the probability that a randomly selected household speaks only English or only Spanish at home?
 - (b) What is the probability that a randomly selected household speaks a language other than only English at home?
 - (c) What is the probability that a randomly selected household speaks a language other than only English or only Spanish at home?
 - (d) Can the probability that a randomly selected household speaks only Polish at home equal 0.08? Why or why not?
6. The packaging of an E.P.T. Pregnancy Test states that the test is “99% accurate at detecting typical pregnancy hormone levels.” Assume that the probability that a test will correctly identify a pregnancy is 0.99 and that 12 randomly selected pregnant woman with typical hormone levels are each given the test.
- (a) What is the probability that all 12 tests will be positive?
 - (b) What is the probability that at least one test will not be positive?
7. A random number generator generates numbers between 0 and 10. Let random variable X be the number generated. Suppose X has a uniform distribution. What is the probability that the computer generates a number between 1 and 4? Note: You must find the probability density function of X .
8. A standard 52-card deck contains four 10's and twelve face cards (jacks, queens and kings). What is the probability of getting two 10's and three face cards and a 5-card poker hand? Note: You must explain your work through words and/or formula(s).
9. An advisory committee at a university requires six members to be chosen from 20 equally qualified people as follows. First a president and a vice-president of the committee are chosen and then four additional members, of equal standing, are chosen from the remaining 18 people. How many advisory committees of this type can be selected? Show your work.
10. A basketball player can make 80% of her free throws. Suppose she attempts 5 free throws, what is the probability that misses one or more free throws? Note: You must explain your work through words and/or formula(s).
11. Suppose an AutoZone store receives a shipment of size 100 alternators from the manufacturer. The store manager checks 3 alternators in random and if one or more are defective, then shipment is rejected. If there are four defective alternators in the shipment, what is the probability of the shipment being rejected? Note: You must explain your work through words and/or formula(s).

12. It costs \$5 to play a dice game. In this game you roll a fair six-sided die. If you roll a 1, 2, or 3 you will be paid \$1. If you roll a 4 or 5, you will be paid \$2. And if you roll a 6 you will be paid \$11. What is your expected payoff in this game? If you play this game a lot, do you expect to make or lose money? Why? Note: State the appropriate formula and show your work.
13. The birth weights of full-term babies are normally distributed with $\mu = 3,400$ grams and $\sigma = 505$ grams. What is the probability a full-term baby weighs more than 4,410 grams? What percentage of babies will weight between 2,794 grams and 3,703 grams?
14. Studies show that gasoline use for compact cars sold in the United States is normally distributed, with mean of 25.5 miles per gallon (mpg) and a standard deviation of 4.5 mpg. What percentage of cars have an mpg of 33.69 or higher?

Just Answers To Additional (not on the Sample Exam) Review Problems for Chapters 5-7

The following are just answers, not complete solutions!

1. (a) event (b) $P(E) + P(F)$ (c) 56 (d) $1/8$ (e) $P(E)P(F)$ (f) 0.3 (g) 0.4013
2. (a) False (b) True (c) True (d) True (e) True
3. (a) $7! = 5040$ (b) ${}_7C_5 = 21$ (c) ${}_7P_3 = 210$
4. (a) 15 (b) 12% (c) 15%
5. (a) $P(\text{only English or only Spanish}) = 0.93$ (b) $P(\text{other than English}) = 0.19$
(c) $P(\text{other than only English or only Spanish}) = 0.07$ (d) No, since $0.08 > 0.07$.
6. (a) $P(\text{All 12 positive}) = 0.99^{12} = 0.8864$ (b) $P(\text{At least one not positive}) = 0.1136$
7. $P(\text{Number between 1 and 4}) = \frac{1}{10-0} \times (4 - 1) = 0.3$
8. $P(\text{two 10's and three face cards}) = \frac{{}_4C_2 \times {}_{12}C_3}{{}_{52}C_5} \approx 0.0005$
9. ${}_{20}P_2 \times {}_{18}C_4 = 1,162,800$
10. (a) $P(\text{making all free throws}) = 0.8^5$, $P(\text{missing one or more free throws}) = 1 - 0.8^5 = 0.67232$
11. $P(\text{All three not defective}) = \frac{{}_{96}C_3}{{}_{100}C_3}$, $P(\text{Rejection}) = 1 - \frac{{}_{96}C_3}{{}_{100}C_3} \approx 0.1164$
12. Expected payoff = $\frac{3}{6} \times \$1 + \frac{2}{6} \times \$2 + \frac{1}{6} \times \$11 = \3 ; Lose money.
13. Let X be the weight of babies. $P(X > 4410) = P(Z > 2) = 0.0228$, $P(2794 < X < 3703) = P(-1.2 < Z < 0.6) = 0.6106 \approx 61\%$.
14. Let X be the mpg. $P(X > 33.69) = P(Z > 1.82) = 0.0344 \approx 3\%$.